Truth Tables

$$\neg(P \text{ and } Q) \iff \neg P \text{ or } \neg Q$$

logically equivalent

TEST

P	Q	$\neg P$	$\neg Q$	P and Q	$\neg(P \text{ and } Q)$	$\neg P$ or $\neg Q$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

$$[\neg(P \text{ and } Q) \iff \neg P \text{ or } \neg Q]$$

T

T

T

Truth Tables

T

Compound statements can be complicated and Truth Tables let you calculate with them.

An example

Professor says: If you get an A on the final, or you get at least 90 on the homework, then you pass this course.

This statement is TRUE provided that the Professor told the truth (didn't lie) – whether or not you get an A in the course.

Analysis

- You ~~get an A~~ Pass in this course (P)
- You get an A on the final (Q)
- You get at least 90 on the homework (R)

The promise is:

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If (Q or R) then P.
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How many possibilities?

P	Q	R	Q or R	(Q or R) $\Rightarrow$ P
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	T

Truth Table

$$(Q \vee R) \implies P$$

Another example (see the text, Ch2.5)

Let P and Q be any statements. $(P \vee Q) \wedge \sim(P \wedge Q)$ reads as:

$(P$ OR $Q)$ and NOT $(P$ AND $Q)$. P or Q but not both

		X		Y	
P	Q	$P \vee Q$	$P \wedge Q$	$\sim(P \wedge Q)$	$X \wedge Y$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

$\neg(P \Leftrightarrow Q)$

Example

$$P \iff (Q \vee R)$$

- $xy = 0$ if and only if $x = 0$ or $y = 0$.
- You will pass this course if and only if you get an A on the final or at least 90 on the homework.

P	Q	R	Q or R	(Q or R) $\Rightarrow$ P	(Q or R) $\Leftrightarrow$ P
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	F	T	T

Homework example

Write a truth table for $(P \wedge \sim P) \vee Q$.

F

$$(P \wedge \sim P) \vee Q \iff Q$$

$$\underline{(P \wedge \sim P)}\ \cancel{\vee}\ Q \iff \text{"False"}$$

$\wedge$